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Construction of 16-QAM OFDM Codes using QPSK Golay Sequences with Large Euclidean Distance and Reduced PAPR

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***Abstract*—This paper is dedicated to the reduction of Peak to Average Power Ratio (PAPR) in the prevailing multicarrier transmission scheme of Orthogonal Frequency Division Multiplexing (OFDM) by recognizing 16-QAM as set addition of two QPSK constellations. As high PAPR is one of the major impediments to deploying OFDM, Golay complementary sequences over vector space Z_4 are used to construct 16-QAM OFDM sequences with low PAPR. Five 16-QAM OFDM sequences with low PAPR and large squared Euclidean distance are analyzed. Further the manuscript examines the code rate and the squared Euclidean distance of these sequences. It has been investigated that there is a tradeoff between squared Euclidean distance and code rate with increasing number of subcarriers in OFDM system.**

***Index Terms*—Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Golay Complementary Sequences (GCS), Squared Euclidean distance, code rate**

I. INTRODUCTION

OFDM is one of the most popular multicarrier modulation schemes and was patented in the United States in 1970. OFDM has emerged as the standard of choice in many high data applications due to its inherent robustness against multipath fading channels, easy implementation and high spectral efficiency [1]. OFDM-based systems are more immune to impulse noise, fast fades and can be of greater pursuit for wireless applications as they eliminate the need for highly sophisticated equalizers. A major barrier to the widespread acceptance of OFDM is the high peak-to-average power ratio (PAPR) of uncoded signals. A higher PAPR means the system requires large linearity range for the analog circuitry used and also increase in the dynamic range of the analog-to-digital converter. PAPR reduction methods can be categorized as distortion type and non-distortion type techniques. Clipping [2], Peak Windowing, Peak Cancellation [3] methods are some examples of distortion type techniques whereas non-distortion type techniques consists of Peak Reduction Carrier, Selective Level Mapping (SLM) [4], Partial Transmit Sequence (PTS) [5] and Block Coding techniques. Coding method not only reduces PAPR to greater extent but also has the advantage of error correction associated with it. Golay codes are an excellent example of coding method whose PAPR is upper bound by 2 for any Golay sequence in PSK modulation [6]

Davis and Jedwab disclosed a relationship between Golay Complementary Sequences (GCS) and second order Reed-Muller (RM) codes that the 2^h-PSK Golay sequences can be obtained from certain cosets of the first order Reed-Muller codes. In other words the large sets of binary length Golay complementary pairs can be obtained from certain second-order cosets of the classical first order RM code.

The rest of the paper is organized as follows. Section II provides an insight into the basic concepts of PAPR and Golay complementary sequences. In section III the construction of 16-QAM OFDM sequences is discussed and its PAPR is calculated. Section IV presents detailed analysis particularly the code rate of different OFDM codes. Finally the paper is concluded in Section V.

II. PRELIMINARIES

A. OFDM

The transmitted OFDM signal can be represented as the real part of the complex signal and is given as

$$S(t) = \sum_{i=0}^{n-1} c_i(t) e^{2j\pi f_i t} \qquad (1)$$

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Where f_i is the frequency of the ith carrier, $c_i(t)$ is constant over a symbol period of duration T. To maintain orthogonality, the carrier frequencies are related by

$$f_i = f_0 + i\Delta f \tag{2}$$

f_0 is the smallest carrier frequency and Δf is the integer multiple of the OFDM symbol rate.

Instantaneous envelope power associated with signal is

$$P_c(t) = \left|S_c(t)\right|^2 = S_c(t).S_c^*(t) \tag{4}$$

Also the average power of c can be expressed as

$$\frac{1}{T}\int_0^T P_c(t)\,dt = \|c\|^2 = \sum_{k=0}^{n-1}|c_k|^2 \tag{5}$$

PAPR is defined as the ratio between the instantaneous power of the peaks and the average power of the signal. Therefore it can be written as

$$\mathrm{PAPR}(c) = \frac{\max\limits_{0\le t\le T} P_c(t)}{P_{av}} \tag{6}$$

B. GOLAY COMPLEMENTARY SEQUENCES

More than sixty years ago, efforts by Marcel Golay led to the discovery of complementary sequences which were later named after him as Golay complementary sequences [7]. Two sequences are said to be Golay complimentary pair (GCP) if sum of their autocorrelation functions vanishes at all delays other than zero. Any sequence which is the member of the Golay complementary (GCP) pair is called a Golay complementary sequence (GCS). The aperiodic autocorrelation of the sequence $c = (c_0, c_1, \ldots, c_{n-1})$ of length n such that $c_i \in \{+1, -1\}$ is defined by

$$A_c(u) = \sum_{i=0}^{n-u-1} c_i . c_{i+u} = c_u \otimes c^*_{-u} \tag{7}$$

The PAPR is upper bound by 2 for any Golay sequence c in PSK modulation.

III. CONSTRUCTION OF 16-QAM OFDM SEQUENCES

Rößing and Tarokh [8], Chong et.al [9] observed and demonstrated that any 16-QAM symbol can be decomposed uniquely into a pair of QPSK symbols as shown in fig.1. So any point on the 16-QAM constellation can be written as

$$S_{16-QAM} = \frac{1}{\sqrt{2}} S_{QPSK} + \sqrt{2} S_{QPSK}$$

Let $c = (c_0, c_1, \ldots, c_{n-1})$, $c_i \in S_{16-QAM}$ be the 16-QAM sequence and can be associated with two QPSK sequences $S_x(t)$ and $S_y(t)$ where $S_{QPSK} = \left\{ e^{\frac{j\pi}{4}}, je^{\frac{j\pi}{4}}, -e^{\frac{j\pi}{4}}, -je^{\frac{j\pi}{4}} \right\}$ So, c_k can be represented as

$$c_k = \left(\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k}\right) e^{j\pi/4} \tag{10}$$

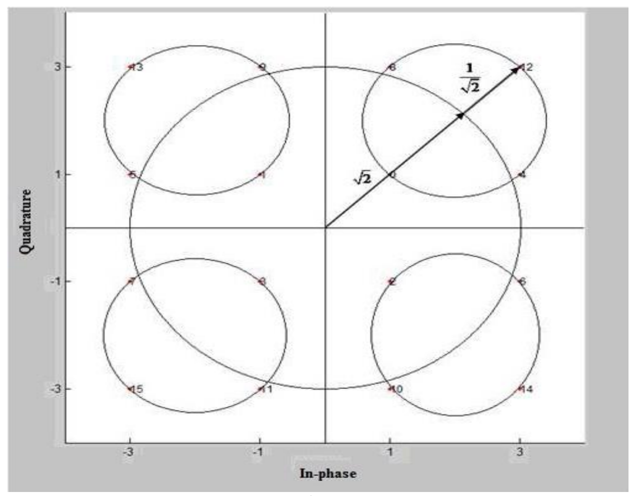

Fig.1 16-QAM as sum of two QPSK symbols

A 16-QAM OFDM signal may be written as the weighted sum of two QPSK OFDM signals as

$$S_c(t) = \sum_{k=0}^{n-1} c_k . e^{\frac{j2\pi kt}{T}}$$

From (10),

$$S_c(t) = \sum_{k=0}^{n-1} \left(\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k}\right) e^{j\pi/4} . e^{\frac{j2\pi kt}{T}}$$

$$\therefore S_c(t) = \frac{1}{\sqrt{2}} S_x(t) + \sqrt{2} S_y(t)$$

The minimum Euclidean distance between signal points of 16-QAM constellation is assumed to be E = 1. Thus for each signal point with equal probability, the average energy of 16-QAM

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constellation $c_k=\sum_{k=0}^{n-1}\left|c_k\right|^2$ is equal to $\left[\sqrt{\left(\frac{1}{2}\right)^2+\left(\sqrt{2}\right)^2}\right]^2=2.5$

Therefore, Average power is $_{av}P$ =2.5 n.Using (4) the instantaneous envelope power is given as

$$P_c(t)=\left|S_c(t)\right|^2=\left|\frac{1}{\sqrt{2}}S_x(t)+\sqrt{2}S_y(t)\right|^2$$

If two QPSK Golay sequences of length 2^m are used as the component codes, the resulting 16-QAM code c of length $n = 2^m$ has PAPR bounded by

$$PAPR(c)\le\frac{\left|\frac{1}{\sqrt{2}}.\sqrt{2n}+\sqrt{2}.\sqrt{2n}\right|^2}{2.5n}=\frac{9}{2.5}=3.6$$

Therefore PAPR for 16-QAM OFDM signal is bounded above by 3.6 [10].

IV. ANALYSIS OF 16-QAM CODES

For a 16-QAM OFDM code we can trade off the size, the PAPR, and the squared Euclidean distance of an OFDM code by an appropriate selection of two QPSK component codes. This section presents and analyzes different codes along with their BPSK component codes.

A. CODE Z_1

Let 16-QAM code Z_1 be decomposed into four BPSK codes which can be represented by B_1, B_2, B_3, B_4. From the values in table it is conjectured that $B_1=B_3=RM_2(1,m)$ and $B_2=B_4=Q_\Pi+RM_2(1,m)$ where Q_Π represents all $\frac{m!}{2}$ second order Boolean polynomials of the form $Q_\pi=x_{\pi(1)}x_{\pi(2)}+\ldots+x_{\pi(m-1)}x_{\pi(m)}$ and $Q=x_1x_2+\ldots+x_{m-1}x_m$ as one of them [10]. The size of code Z_1 of length $n=2^m$ is $\left(\frac{m!}{2}\right)^2\left(4^{m+1}\right)^2$.

The code rate (R) for BPSK codes associated with the QPSK code Z_1 can be calculated for different values of n as,

Code rate $R(C)=\frac{\log_2|C|}{n}$ bits/symbol, where, n is the length of the code and $|Z_1|=\left(\frac{m!}{2}\right)^2\left(4^{m+1}\right)^2$ [10]

For n=16, m=4, $\log_2|Z_1|=27.168$, $R(Z_1)=\frac{27.168}{16}=1.698$

For n=32, m=5, $\log_2|Z_1|=35.808$, $R(Z_1)=\frac{35.808}{32}=1.119$

For n=64, m=6, $\log_2|Z_1|=44.992$, $R(Z_1)=\frac{44.992}{64}=0.703$

For n=128, m=7, $\log_2|Z_1|=54.528$, $R(Z_1)=\frac{54.528}{128}=0.426$

For n=256, m=8, $\log_2|Z_1|=64.512$, $R(Z_1)=\frac{64.512}{256}=0.252$

The minimum squared Euclidean distance between two sequences is given by the expression

$$D_E^2=\min\left(d_i,a_i,E^2\right) \qquad (11)$$

Where d_i is the minimum Hamming distance of the associated binary code, $a_i=2^{i-1},\ 1\le i\le 4$

So for code Z_1 the minimum squared Euclidean distance is $D_E^2=\min\left(1.2^{m-1},2.2^{m-2},4.2^{m-1},8.2^{m-2}\right)=2^{m-1}$

B. CODE Z_2

Let code Z_2 be decomposed into four BPSK codes shown in TABLE I. The size of code Z_2 of length $n=2^m$ is given by

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$\left|Z_2\right| = \left(\frac{m!}{2}\right)\left(4^{m+1}\right)\left(2^{m+2}\right)$. In the similar manner as for code Z_1 the code rate for Z_2 can be calculated for different values of *n*. The calculated values are tabulated in TABLE V. The resulting 16-QAM code has $D_E^2 = \min\left(1.2^m, 2.2^{m-1}, 4.2^{m-1}, 8.2^{m-2}\right) = 2^m$ and $PAPR(Z_2) = 3.6$ as shown previously.

TABLE I 16-QAM OFDM CODES WITH PAPR=3.6[10]

	Z_1	Z_2
B_1	$RM(1,m)$	$RM(0,m)$
B_2	$Q_\Pi + RM(1,m)$	$Q + RM(1,m)$
B_3	$RM(1,m)$	$RM(1,m)$
B_4	$Q_\Pi + RM(1,m)$	$Q_\Pi + RM(1,m)$
size	$\left(\frac{m!}{2}\right)^2\left(4^{m+1}\right)^2$	$\left(\frac{m!}{2}\right)\left(4^{m+1}\right)\left(2^{m+2}\right)$
PAPR	≤ 3.6	≤ 3.6
D_E^2	2^{m-1}	2^m

TABLE II 16-QAM OFDM CODES WITH PAPR 5.86 [10]

	Z_3
B_1	$RM(0,m)$
B_2	$Q + RM(1,m)$
B_3	$RM(1,m-1) \oplus RM(1,m-1)$
B_4	$Q_\Pi + RM(1,m-1) \oplus Q_\Pi + RM(1,m-1)$
size	$\left(\frac{(m-1)!}{2}\right)^2 (2)^{m+2}\left(4^m\right)^2$
PAPR	≤ 5.86
D_E^2	2^m

C. CODE Z_3

TABLE III 16-QAM OFDM CODES WITH PAPR=5.86 [10]

	Z_4
B_1	$RM(1,m)$
B_2	$Q_\Pi + RM(1,m)$
B_3	$RM(1,m-1) \oplus RM(1,m-1)$
B_4	$Q_\Pi + RM(1,m-1) \oplus Q_\Pi + RM(1,m-1)$
Size	$\frac{m!}{2}\left(\frac{(m-1)!}{2}\right)^2 (4)^{m+1}\left(4^m\right)^2$
PAPR	≤ 5.86
D_E^2	2^{m-1}

TABLE IV 16-QAM OFDM CODES WITH PAPR=10 [10]

	Z_5
B_1	$RM(1,m)$
B_2	$Q_\Pi + RM(1,m)$
B_3	$RM(1,m-2) \oplus RM(1,m-2)$ $\oplus RM(1,m-2) \oplus RM(1,m-2)$
B_4	$Q_\Pi + RM(1,m-2) \oplus Q_\Pi + RM(1,m-2)$ $\oplus Q_\Pi + RM(1,m-2) \oplus Q_\Pi + RM(1,m-2)$
size	$\frac{m!}{2}\left(\frac{(m-2)!}{2}\right)^4 (4)^{m+1}\left(4^{m-1}\right)^4$
PAPR	≤ 10
D_E^2	2^{m-1}

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Let code Z_3 be decomposed into BPSK sequences presented in TABLE II. It is known that for any two given codes $L_2(n_2,M_2,d_2)$ and $L_2(n_2,M_2,d_2)$, the direct sum of L_1 and L_2, denoted by $L_1 \oplus L_2$, is a $(n_1+n_2, M_1M_2, \min(d_1,d_2))$ code. Using this analogy for codes B_3 and B_4 we get the size of code Z_3 as, $|Z_3| = \left(\frac{(m-1)!}{2}\right)^2 (2)^{m+2}\left(4^m\right)^2$. The code rate for Z_3 is calculated as shown above and results are presented in TABLE V. The minimum squared Euclidean distance is $D_E^2 = \min\left(1.2^m, 2.2^{m-1}, 4.2^{m-2}, 8.2^{m-3}\right) = 2^m$. Since B3 and B4 are second order Reed-Muller codes

so $PAPR(Z_3) = \dfrac{(\frac{1}{\sqrt{2}}\sqrt{2n}+\sqrt{2}\sqrt{4n})}{2.5n} = 5.68$

D. CODE Z_4

The code Z_4 is decomposed into four BPSK codes shown in TABLE III. Size of code Z_4 is given as $|Z_4| = \frac{m!}{2}\left(\frac{(m-1)!}{2}\right)^2 (4)^{m+1}\left(4^m\right)^2$. The minimum squared Euclidean distance is calculated as $D_E^2 = \min\left(1.2^{m-1}, 2.2^{m-2}, 4.2^{m-2}, 8.2^{m-3}\right) = 2^{m-1}$. Just as in the case of code Z_3 PAPR of code Z_4 can also be calculated as

$$PAPR(Z_4) = \frac{(\frac{1}{\sqrt{2}}\sqrt{2n}+\sqrt{2}\sqrt{4n})}{2.5n} = 5.68$$

E. CODE Z_5

The code Z_5 is decomposed into four BPSK codes shown in TABLE IV. whose size is $|Z_5| = \frac{m!}{2}\left(\frac{(m-2)!}{2}\right)^4 (4)^{m+1}\left(4^{m-1}\right)^4$. The code rate for Z_5 can be calculated as for the case of Z_1. The minimum squared Euclidean distance $D_E^2 = \min\left(1.2^{m-1}, 2.2^{m-2}, 4.2^{m-2}, 8.2^{m-3}\right) = 2^{m-1}$

TABLE V COMPARISON BETWEEN 16-QAM OFDM CODES [10]

Code		Z_1	Z_2	Z_3	Z_4	Z_5
PAPR		3.6	3.6	5.86	5.86	10
n=16	D_E^2	8	16	16	8	8
	R	1.698	1.224	1.573	2.047	2.350
n=32	D_E^2	16	32	32	16	16
	R	1.119	0.778	1.068	1.409	1.758
n=64	D_E^2	32	64	64	32	32
	R	0.703	0.476	0.685	0.911	1.201
n=128	D_E^2	64	128	128	64	64
	R	0.426	0.284	0.422	0.565	0.773
n=256	D_E^2	128	256	256	128	128
	R	0.252	0.166	0.252	0.340	0.478

Also, the PAPR is bounded by

$$PAPR(Z_5) = \frac{(\frac{1}{\sqrt{2}}\sqrt{2n}+\sqrt{2}\sqrt{8n})}{2.5n} = 10$$

For the ease of understanding and comparison, the above results are tabulated in TABLE V. and using these values graphs of Euclidean distance and code rate are plotted in Fig.2 and Fig.3.

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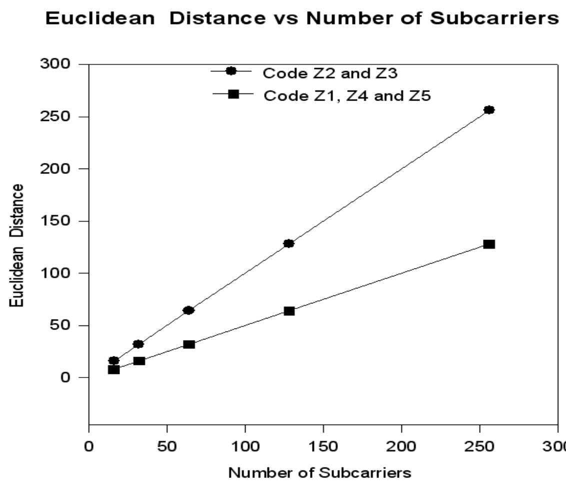

Fig.2 Plot for Euclidean Distance

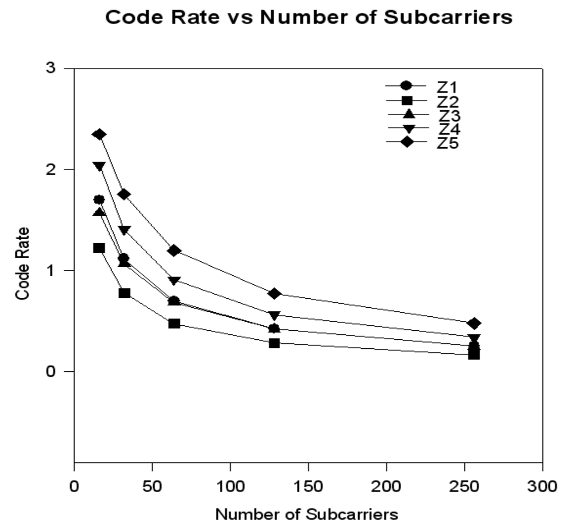

Fig.3 Plot for Code Rate

V. CONCLUSION

From Fig.2 it is conjectured that the Euclidean distance for code Z_2 and Z_3 is large thereby providing a better BER performance and error correction capability. Also Fig.3 depicts that code rate performance for code $Z_5 \geq Z_4 \geq Z_1 \geq Z_3 \geq Z_2$. But if we consider the PAPR level which is the most important parameter then code Z1 and Z2 are best since they have least value of PAPR among all five codes. Code Z5 has the best code rate performance but provides very poor PAPR reduction. So, after the overall analysis of all the parameters it is concluded that for sequences with smaller length i.e $n \leq 64$ code *Z*3 has comparable rate, larger Euclidean distance and thereby better error correction capability as compared to the code *Z*1. But Z1 has lower PAPR than Z3. Also, codes Z4 and Z5 have average PAPR level and same Euclidean distance or error correction capability as Z1 but possesses much higher code rate than Z1. So they can be used in systems supporting larger length sequences i.e. $n \geq 128$. So, the tradeoff between PAPR, the code rate, and the squared Euclidean distance can be achieved by wisely choosing the appropriate code.

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